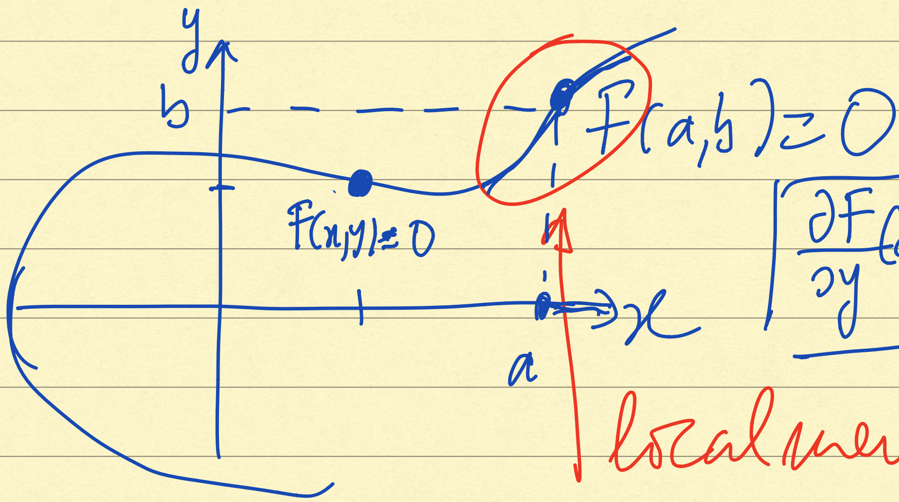
Inversa. Implícita

$F: \mathbb{R}^2 \to \mathbb{R}$, C^1, $F(a,b) = 0$



$$\boxed{\frac{\partial F}{\partial y}(a,b) \neq 0}$$

localmente

$$F(x,y) = 0 \Leftrightarrow y = f(x)$$

implícita

$$F(a, f(x)) = 0$$

$$\Rightarrow \frac{\partial F}{\partial x}(a,b) + \frac{\partial F}{\partial y}(a,b)\underbrace{f'(a)}_{\text{Derivada da implícita}} = 0$$

Truque: $G(x,y) = (F(x,y),\ x)$

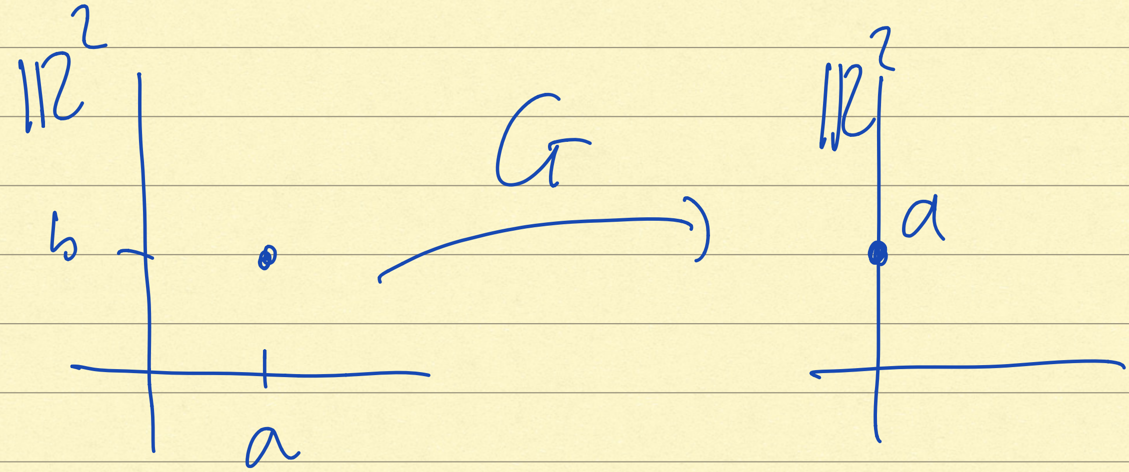
$y = f(x)$ — x livre

y dependente

$$\begin{cases} F(x,y) = 0 \\ x = x \end{cases}$$

$$G : \mathbb{R}^2 \longrightarrow \mathbb{R}^2,\ C^1$$

$$G(x,y) = (0, x)$$



Se $\boxed{G^{-1} \text{ existir}}$

$$G(x,y) = (0,x)$$

$$\iff (x,y) = G^{-1}(0,x)$$

$$(x,y) = \underbrace{G^{-1}(0,x)}_{x}$$

y é função de x.!!!

$$G : \mathbb{R}^2 \longrightarrow \mathbb{R}^2, \quad C^1$$

$$G(a,b) = (F(a,b), a) = (0,a)$$

$$DG(a,b) = \begin{bmatrix} \frac{\partial F}{\partial x}(a,b) & \frac{\partial F}{\partial y}(a,b) \\ 1 & 0 \end{bmatrix}$$

2×2

$$\boxed{\det DG(a,b)} = -\underbrace{\frac{\partial F}{\partial y}(a,b)}_{} \neq 0$$

$G : \mathbb{R}^n \longrightarrow \mathbb{R}^n$, linear.

$G(x) = Ax$, $A_{n \times n}$

$G(x) = y \iff x = G^{-1}(y)$

$\boxed{\det A \neq 0}$

$\det A = \boxed{\det DG(x)} \neq 0$

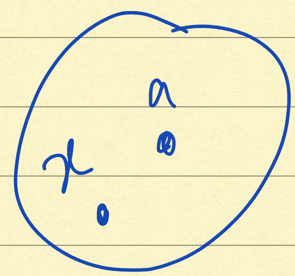
$G : \mathbb{R}^n \longrightarrow \mathbb{R}^n$, C^1

$\underbrace{G(x) - G(a)} = DG(a)(x-a) + o(x-a)$

$G(a) = 0$

$$G(x) = DG(a)(x-a) + \underbrace{o(x-a)}_{\approx 0}$$

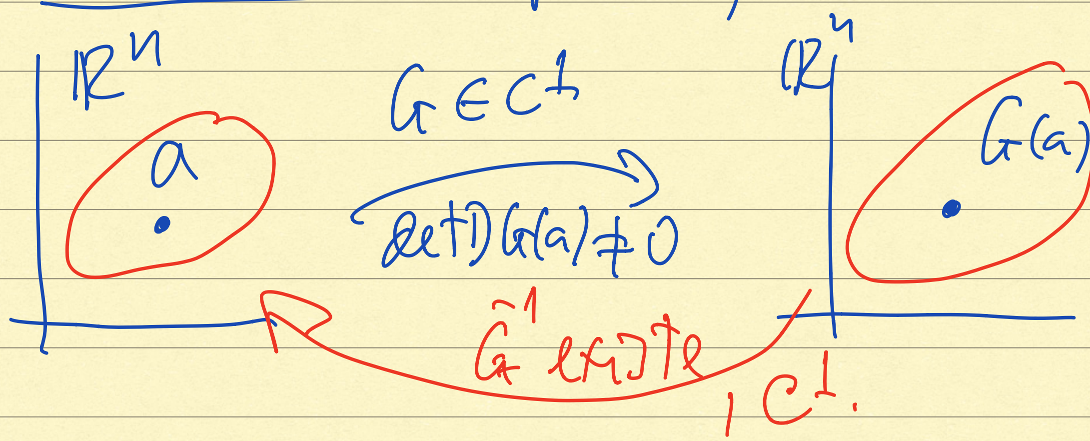
local.



$$G(x) \approx \underbrace{DG(a)(x-a)}_{\text{linear}}$$

$$\boxed{\boxed{\det DG(a) \neq 0} \implies \boxed{G^{-1} \text{ există, } G^{-1} \in C^1.}}$$

Teorema de funcție inversă.



$$\begin{tikzcd} x \arrow[r, "G"] & G(x) = y \arrow[l, bend left, "G^{-1}"] \end{tikzcd}$$

$$x = G^{-1}\big(G(x)\big)$$

$$I = DG^{-1}\big(G(x)\big)\,\underbrace{DG(x)}$$

$$\boxed{DG^{-1}\big(\underbrace{G(x)}_{y}\big) = \big(DG(x)\big)^{-1}}$$

$$F(x,y) = 0 \overset{??}{\iff} y = f(x)$$

localmente

f existe C^1.

$$G(x,y) = \big(F(x,y), x\big) = (0, x)$$

$$G(a,b) = (0, a)$$

$$\det DG(a,b) = -\frac{\partial F}{\partial y}(a,b) \neq 0$$

$$\Rightarrow \boxed{G^{-1} \text{ existe}, \ C^1}.$$

$$G(x,y) = (0, x)$$

$$\iff (x,y) = \underbrace{G^{-1}(0,x)}_{x}$$

Teorema de f. Implícita ($\mathbb{R}^2$)

1) $F : \mathbb{R}^2 \to \mathbb{R}$, C^1.

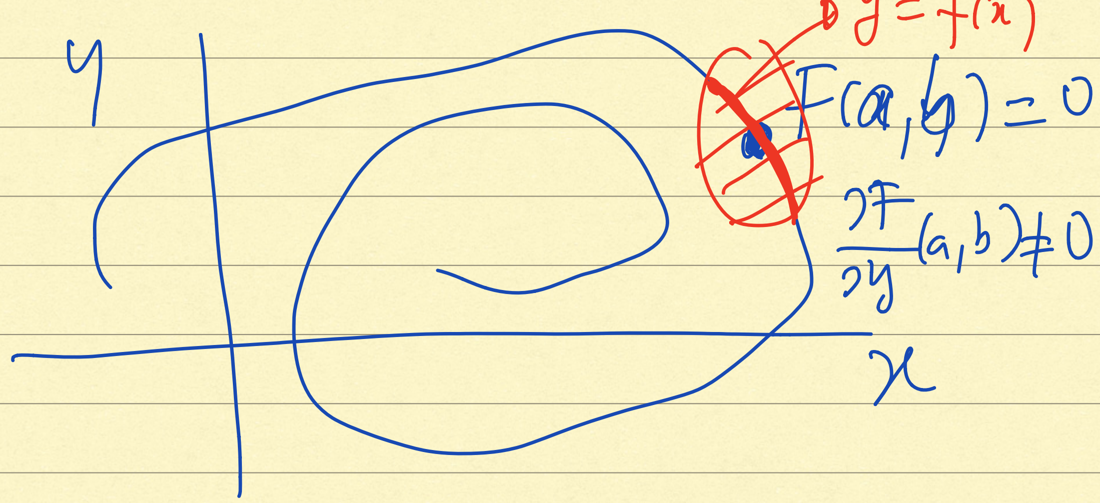
2) $F(a,b) = 0$

3) $\dfrac{\partial F}{\partial y}(a,b) \neq 0$

$\Longrightarrow F(x,y) = 0 \overset{\text{localmente}}{\Longleftrightarrow} y = f(x)$

existe, C^1

$f : \mathbb{R} \to \mathbb{R}$

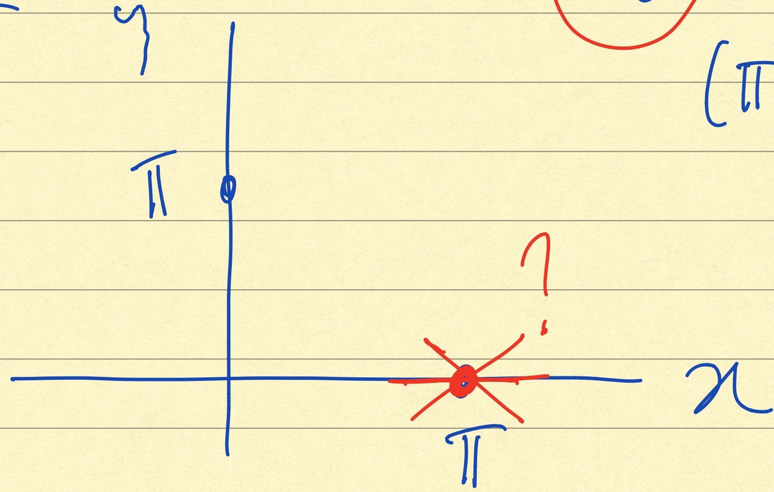


$$\boxed{\frac{\partial F}{\partial x}(a,b) + \underbrace{\frac{\partial F}{\partial y}(a,b)}_{\neq 0}\, \underbrace{f'(a)}_{} = 0}$$

$$\text{———————— } \| \text{ ————————}$$

<u>Ex</u>: $\sin(x+y) + xy = 0$

$(\pi, 0)$



$$\left[\cos(x+y) + y \qquad \cos(x+y) + x\right]$$

$x = h(y)$ $\qquad$ $y = f(x)$ $\qquad$ $(\pi, 0)$

$$= \left[\underbrace{-1}_{\neq 0} \neq 0 \qquad \underbrace{-1 + \pi}_{\neq 0}\right]$$

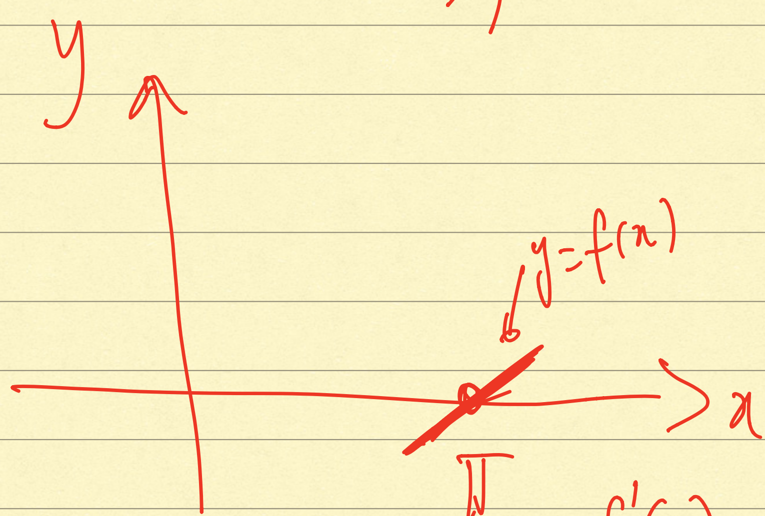
$y = f(x)$ $\longrightarrow$ $F(x, f(x)) = 0$

$(x_1) \in (e, c)$

$$f'(a) = - \frac{\frac{\partial F}{\partial x}(a,b)}{\frac{\partial F}{\partial y}(a,b)}$$

$$f'(\pi) = - \frac{\frac{\partial F}{\partial x}(\pi,0)}{\frac{\partial F}{\partial y}(\pi,0)} = - \frac{-1}{\pi - 1} = \boxed{\frac{1}{\pi - 1}}$$

> 0



$f'(\pi) > 0$

$$F(x,y)=0$$

$$DF(a,b)=\left[\frac{\partial F}{\partial x}(a,b) \quad \underbrace{\frac{\partial F}{\partial y}(a,b)}_{\neq 0}\right]$$

$$f'(a) = -\frac{\frac{\partial F}{\partial x}(a,b)}{\frac{\partial F}{\partial y}(a,b)}$$